Perceptual Learning in Mathematics Education

**Units and Fractions**

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### Perceptual Learning (PL)

- **Experience-inducement of changes in the ability to extract information**
- **Selectively perceive problem-relevant features as well as higher-level relationships**
- **Evenly instructed about higher-level relationships, novices often ignore them or cannot recognize them in new cases**
- **PL is crucial in the development of expertise but largely neglected in K-12 instruction**

#### Current Research Questions:

1. Can PL be systematically accelerated in mathematics – in particular, in the domains of fractions and measurement?
2. Can PL interventions (short varying interactions) improve performance on traditional assessments?

#### Learning Goal: Fractions as Quantities

- Students have limited interpretations of fractions, e.g., the common part-whole interpretation (3 out of 5 slices of a pizza) is hard for students to generalize to improper and equivalent fractions, and to operations with fractions.

- **1 unit**
- **5/6 units**
- **7/5 or 7/5 units?**

The two modules presented here (Slice&Clone 1 and 2) are designed to help students come to understand fractions as new quantities, in which the denominator of the fraction defines an iterable unit and the numerator of the fraction specifies a given number of iterations of that unit.

#### Design & Participants

- **Control participants (7th graders)** completed the most relevant fraction/measurement units taught in their curriculum before testing (7 weeks)
- **PLM participants (6th graders)** received all treatment & tests except delayed posttest prior to these curriculum units.

#### Slice & Clone 1

- **The learning focus:** creating and iterating unit fractions to produce new quantities (integers, fractions, and mixed numbers with the same denominator).

- Each trial involves one parent quantity (given) and one offspring quantity (to be created).

- User controls interactive “slicing” and “cloning” tools.

- **Variation in Trial Difficulty**
  - **1. Parent/offspring value:** may be integer (1 or >1) or fraction (<1, improper, or mixed fraction)
  - **2. # of conceptual steps:**
    - **One-step:** No chunking to slice or clone.
    - **Two-steps:** The cloning of the offspring requires chunking to create one whole out of 1/5 units; then adding on another three 1/5 units.

#### Slice & Clone 2

- **The learning focus:** equivalent fractions involving different denominators

- Each trial involves one parent quantity (given) and two different offspring quantities (to be created).

- **Variation in Trial Difficulty**
  - **1. Parent/offspring value:** may be integer (1 or >1) or fraction (<1, improper, or mixed fraction)
  - **2. # of conceptual steps:**
    - **One-step:** No chunking to slice or clone.
    - **Two-steps:** The cloning of the offspring requires chunking to create one whole out of 1/5 units; then adding on another three 1/5 units.

### Results

#### Criteria for completion of S&C 1 PLM

- 11 categories (one-step/low difficulty: 4 categories; two-step/high difficulty: 7 categories); 8/10 correct in each category
- 100% learned to criterion in the 11 categories of SC1

#### Average # of categories learned to criterion: 11

- Criteria for completion of S&C 2 PLM
  - 8 categories (5 active slice/done categories; 3 passive slice/active done categories); 8/10 correct in each category
  - 70% learned to criterion in the 8 categories of SC2
  - Average # of categories learned to criterion: 6.8

#### Effect sizes

- **Posttest vs. Pretest:** 2.03
- **Delayed Posttest vs. Pretest:** 1.75
- **Posttest vs. Control:** 3.45
- **Delayed Posttest vs. Control:** 2.79

#### Conclusions

- **PL technology can facilitate structure discovery and structure mapping, offering an important complement to declarative and procedural instruction**
- **PL technology can accelerate fluency in processing key structures, reducing cognitive load and facilitating more advanced problem solving**
- **PL techniques show great promise even in high-level, symbolic domains such as mathematics — including notoriously difficult areas such as fractions and measurement**

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